International Journal of Thermophysics, Vol. 12, No. 3, 1991

Simultaneous Measurement of Thermal Conductivity and Thermal Diffusivity of Solids by the Parallel-Wire Method

Y. Z. Zhang,[1] S. X. Cheng,[1] J. A. Lee,[1] and X. S. Ge[1]

Received August 14, 1990

An improved parallel-wire technique for simultaneous measurement of thermal conductivity and thermal diffusivity is presented. The deviation between experimental results and recommended (or another author's) values is less than 5% for fused quartz and refractory brick.

KEY WORDS: hot-wire technique; parallel-wire technique; thermal conductivity; thermal diffusivity.

1. INTRODUCTION

It is well known that the hot-wire method has been used to measure the thermal conductivity of some materials. De Boer et al. [1] modified the technique. They no longer have the measurement thermocouple welded to the midpoint of the heater but place the thermocouple parallel to the heater wire at a distance of about 15 mm. The temperature rise of the test material is detected by the thermocouple rather than the "hot wire," and therefore it is possible to raise the conductivity measurement limit from 2.0 to 25 $W \cdot m^{-1} \cdot K^{-1}$.

Donati et al. [2] developed a two-linear parallel-probe method. According to the technique, two parallel holes are drilled in the sample, inside which two probes, A and B, are inserted. Probe A is the heater wire, as before, but probe B contains only thermistors. This technique can be used to measure the thermal conductivity and the thermal diffusivity of nonhomogeneous materials simultaneously.

[1] Department of Engineering Thermophysics, University of Science and Technology of China, Hefei, Anhui 230026, People's Republic of China.

840/12/3-10

A modification presented in this paper lies in replacing the thermocouple or the thermistors by another wire similar to the heater wire. This method permits the measurement of the average temperature rise along the length of the wire, instead of the "spot" measurement provided by the welded thermocouple. It also provides the advantage of easier self-checking of the obtained measurements as compared with the thermistor method developed by Donati [2]. In fact, the self-checking becomes very simple because of the exchangeability of the two similar wires or using the single-wire method (i.e., either of the wires does not work for the same material).

Of course, as a temperature sensor, the metal wire is very different from a thermistor. Therefore, it is necessary to derive a new theoretical model. Results show that the deviation between the experimental results based on this model and the recommended (or another author's) values is less than 5% for fused quartz and refractory brick.

2. THEORETICAL MODEL

Two sufficiantly long parallel metal wires, such as pure platinum or pure nickel wire, are embedded in a test material which can be considered as an infinite medium. If steady currents I_1 and I_2 are supplied to the wires, respectively, the temperature rise ϑ at time t at point (x, y) is expressed by the following differential equation [3, 4]:

$$\lambda\left(\frac{\partial^2\vartheta}{\partial x^2}+\frac{\partial^2\vartheta}{\partial y^2}\right)+Q_1\delta(x, y)+Q_2\delta(x-x_0, y-y_0)=\rho C_P\frac{\partial\vartheta}{\partial t}$$

$$\vartheta(x, y, t)=0, \qquad t=0 \tag{1}$$

$$\vartheta(x, y, t)=0, \qquad x^2+y^2\Rightarrow\infty$$

where λ, α, ρ, and C_P are the thermal conductivity, thermal diffusivity, mass density, and specific heat, respectively; $\delta(x, y)$ and $\delta(x-x_0, y-y_0)$ are the Dirac δ functions at points O and p, respectively; and Q_1 and Q_2 are the quantities of heat being instantaneously generated at points $O(0, 0)$ and $p(x_0, y_0)$ by I_1 and I_2 separately. That is,

$$Q_1=I_1^2R_1$$

$$Q_2=I_2^2R_2$$

where R_1 and R_2 are the resistances of the wires.

Here we suppose that one wire passes through point $O(0, 0)$, the other passes through $p(x_0, y_0)$, and they are both parallel to the Z axis. Obviously, Eq. (1) is the sum of the following two equations:

$$\lambda\left(\frac{\partial^2\vartheta_1}{\partial x^2}+\frac{\partial^2\vartheta_1}{\partial y^2}\right)+Q_1\delta(x, y)=\rho C\frac{\partial\vartheta_1}{\partial t}$$

$$\vartheta_1(x, y, t)=0, \qquad t=0 \tag{2}$$

$$\vartheta_1(x, y, t)=0, \qquad x^2+y^2\Rightarrow\infty$$

$$\lambda\left(\frac{\partial^2\vartheta_2}{\partial x^2}+\frac{\partial^2\vartheta_2}{\partial y^2}\right)+Q_2\delta(x-x_0, y-y_0)=\rho C\frac{\partial\vartheta_2}{\partial t}$$

$$\vartheta_2(x, y, t)=0, \qquad t=0 \tag{3}$$

$$\vartheta_2(x, y, t)=0, \qquad x^2+y^2\Rightarrow\infty$$

Therefore,

$$\vartheta=\vartheta_1+\vartheta_2 \tag{4}$$

or

$$\frac{d\vartheta}{d(\ln t)}=\frac{d\vartheta_1}{d(\ln t)}+\frac{d\vartheta_2}{d(\ln t)}$$

where ϑ_1 and ϑ_2 are the solutions of Eqs. (2) and (3), respectively.

It is well known that [5, 6]

$$\frac{d\vartheta_1}{d(\ln t)}=\frac{Q_1}{4\pi\lambda}\exp\left(-\frac{r_1^2}{4\alpha t}\right)$$

$$\frac{d\vartheta_2}{d(\ln t)}=\frac{Q_2}{4\pi\lambda}\exp\left(-\frac{r_2^2}{4\alpha t}\right)$$

where

$$r_1^2=x^2+y^2$$

$$r_2^2=(x-x_0)^2+(y-y_0)^2$$

Let $K=Q_2/Q_1$; then the solution of Eq. (1) is

$$\frac{d\vartheta}{d(\ln t)}=\frac{Q_1}{4\pi\lambda}\left[\exp\left(-\frac{r_1^2}{4\alpha t}\right)+K\exp\left(-\frac{r_2^2}{4\alpha t}\right)\right] \tag{5}$$

In an actual measurement, we take $r_1 = 0$, $r_2 = r_0$; then Eq. (5) becomes

$$\frac{d\vartheta}{d(\ln t)} = \frac{Q_1}{4\pi\lambda}\left[1 + K\exp\left(-\frac{r_0^2}{4\alpha t}\right)\right] \tag{6}$$

where

$$r_0^2 = x_0^2 + y_0^2$$

Equation (6) is the basic formula for measuring λ and α by the parallel-wire method. In fact, it is an extension of the basic formula of the test using a single hot wire. With the rise in measuring temperature of wire 2 at appropriate time intervals and the parameters Q_1, K, and r_0 known, we can obtain λ and α simultaneously through Eq. (6), using the data processing method presented in the following section.

3. EXPERIMENTAL DATA PROCESSING

Two processing methods are used.

(1) The deviation voltage generated in the output of the bridge caused by the temperature rise of wire 2 is input to a microcomputer at a appropriate time interval preset by the user. A curve for ϑ vs $(\ln t)$ is obtained by the microcomputer using a least-squares fitting program. The quantity $[d\vartheta_i/d(\ln t_i)]_{\text{exp}}$ can be obtained from the experimental curve. On the other hand, we can also obtain the theoretical values $[d\vartheta_i/d(\ln t_i)]_{\text{th}}$ using Eq. (6), provided that a pair of values of λ and α is given. There is one pair of λ and α (say, λ_0 and α_0) which makes

$$\text{Sum} = \sum_{i=1}^{n} \left\{\left[\frac{d\vartheta_i}{d(\ln t_i)}\right]_{\text{exp}} - \left[\frac{d\vartheta_i}{d(\ln t_i)}\right]_{\text{th}}\right\}^2$$

minimum. That pair of λ_0 and α_0 is the result desired.

(2) Replacing λ in Eq. (6) with λ_0 obtained by the single hot-wire method (i.e., $K = 0$), and using the least-squares fitting method, we can also obtain α_0.

It is shown that the two methods give almost the same results.

4. MEASURING SYSTEM

A block diagram of the measuring system is shown in Fig. 1. Two parallel platinum wires with a diameter of 0.1 mm are squeezed between

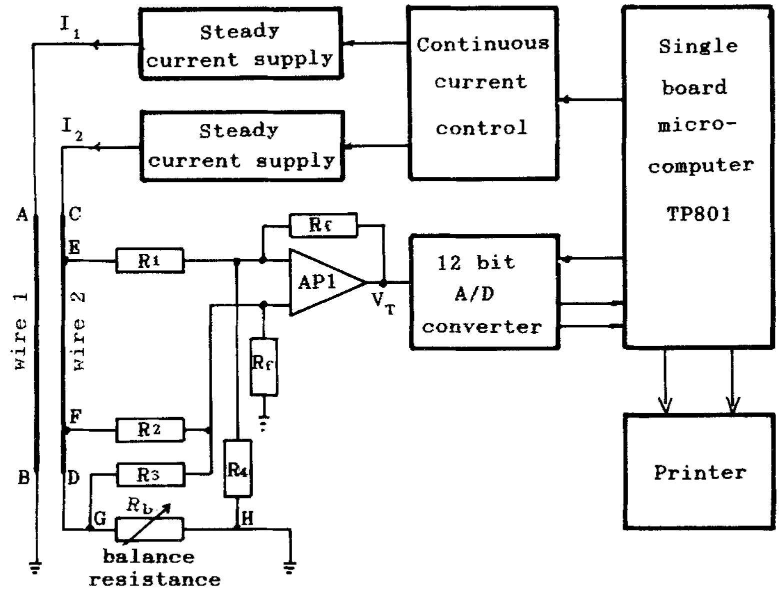

Fig. 1. Block diagram of the measuring system.

two samples. Steady currents I_1 and I_2 are supplied to wires 1 and 2, respectively. Wire 2 forms one arm of a high-precision Kelvin bridge; an adjustable resistance R_b with a very low temperature coefficient forms the other arm (see Fig. 1). Prior to each test run, the bridge had to be balanced by carefully adjusting R_b with only small currents. The temperature rise of wire 2 caused by I_1 and I_2 leads to unbalance of the bridge. The potential difference of the unbalanced bridge is amplified by the amplifier AP1 with high accuracy, high stability, low zero drift, and low noise. Denoting the amplified potential difference as V_T, then

$$\left(\frac{dV_T}{d\ln t}\right)=C\left(\frac{d\vartheta}{d\ln t}\right)$$

where C is a constant dependent on the amplification factor of the amplifier, electrical resistance, and temperature coefficient of resistance of the wire and steady current I_2. V_T is converted into digital signal V_D by a 12-bit A/D converter under the control of a single board microcomputer TP801. Then V_D is printed out or delivered to a computer for further processing.

The steady currents I_1 and I_2 are supplied to wires 1 and 2 by two stabilized current supplies controlled by TP801. The values of I_1 and I_2, time interval, and initial and final time of data acquisition are preset by the user. Of course, in order to obtain highly accurate values of α, the selection of the above-mentioned parameters must satisfy the condition

$$(K)\exp\left(-\frac{r_0^2}{4\alpha t}\right) \sim O(1) \tag{7}$$

In the case of constant K and r_0 (for example, $K=4$, $r_0=4$ mm), αt can be estimated through Eq. (7). Thus the minimum outer diameter R_0 of the sample can be determined by Eq. [7]:

$$R_0 \geqslant 2\sqrt{\alpha t}$$

5. MEASUREMENT RESULTS

To verify the feasibility and estimate the accuracy of the parallel-wire method, the thermal conductivity and thermal diffusivity of fused quartz have been measured. The results are shown in Table I. The comparison of the results obtained by the two above-mentioned processing methods is listed in Table II. The measurement results of the refractory brick are listed in Table III.

In Table I, $\lambda(m)$ and $\alpha(m)$ are the thermal conductivity and thermal diffusivity obtained by data processing method 1, $C(m)$ is the specific heat obtained from the relation $C(m)=\lambda/(\alpha\rho)$, using known $\lambda(m)$, $\alpha(m)$, and $\rho=2200\ \mathrm{kg\cdot m^{-3}}$ (measured ourselves). σ is the standard deviation. The recommended values from TPRC [8] are, respectively,

Table I. The Measured Results of λ and α for Fused Quartz ($T=10°\mathrm{C}$)

Expt. No.	$\lambda(m)$ $(\mathrm{W\cdot m^{-1}\cdot K^{-1}})$	$10^7\alpha(m)$ $(\mathrm{m^2\cdot s^{-1}})$	$C(m)$ $(\mathrm{kJ\cdot kg^{-1}\cdot K^{-1}})$
1	1.25	8.25	0.689
2	1.27	8.05	0.717
3	1.32	8.22	0.730
4	1.31	7.85	0.759
5	1.25	8.25	0.689
6	1.38	8.18	0.767
7	1.32	8.11	0.740
8	1.26	8.24	0.695
9	1.28	8.12	0.717
Ave.	1.29	8.14	0.720
σ	0.04	0.13	0.029

Table II. Comparison of Results Obtained by Two Data Processing Methods for Fused Quartz ($T = 10°C$)

Expt. No.	$\lambda(m)$ $(W \cdot m^{-1} \cdot K^{-1})$	$10^7\alpha(m)$ $(m^2 \cdot s^{-1})$	$10^7\alpha(sf)$ $(m^2 \cdot s^{-1})$
1	1.25	8.25	8.82
2	1.27	8.05	8.58
3	1.32	8.22	8.22
4	1.31	7.85	7.91
5	1.25	8.25	8.11
6	1.38	8.18	8.01
7	1.32	8.11	8.11
8	1.26	8.24	8.91
9	1.28	8.12	8.51
Ave.	1.29	8.14	8.35
σ	0.04	0.13	0.36

$\lambda(rec) = 1.35\ W \cdot m^{-1} \cdot K^{-1}$, $\alpha(rec) = 8.53 \times 10^{-7}\ m^2 \cdot s^{-1}$, and $C(rec) = 0.719\ kJ \cdot kg^{-1} \cdot K^{-1}$. The deviation between the average of measurements and the recommended value is less than 5%.

In Table II, the thermal diffusivities, $\alpha(sf)$, are obtained by processing method 2. The average value of thermal conductivity obtained by the single hot-wire method is $1.32\ W \cdot m^{-1} \cdot K^{-1}$.

It may be noted that the deviation between the average values obtained by the two data processing methods are $\delta\lambda/\lambda = 2.3\%$ and $\delta\alpha/\alpha = 2.6\%$, respectively. The results confirm that the parallel-wire method is a valid method.

In Table III, $C(m)$ is the specific heat of the refractory brick. Its values can also be obtained from the relation as before, given values of $\lambda(m)$, $\alpha(m)$, and $\rho = 2380\ kg \cdot m^{-3}$ (measured ourselves). The average value of the

Table III. Experimental Results for Refractory Brick ($T = 15°C$)

Expt. No.	$\lambda(m)$ $(W \cdot m^{-1} \cdot K^{-1})$	$10^7\alpha(m)$ $(m^2 \cdot s^{-1})$	$C(m)$ $(kJ \cdot kg^{-1} \cdot K^{-1})$
1	1.027	5.25	0.822
2	0.958	5.02	0.802
3	0.963	5.22	0.774
4	1.070	5.71	0.787
Ave.	1.004	5.30	0.796
σ	0.052	0.30	0.0021

specific heat obtained from another apparatus with a maximum error of 2% [9] is $0.760\,kJ \cdot kg^{-1} \cdot K^{-1}$. The relative deviation between them is 4.7%.

It may be concluded that the theoretical model of the parallel-wire method presented here is correct and that the thermal conductivity and thermal diffusivity can be measured simultaneously using this method.

REFERENCES

1. J. de Boer, J. Butter, B. Grosskopf, and P. Jeschke, *Refract. J.* Sept.–Oct.:22 (1980).
2. A. Donati, A. Lanciana, P. Morabito, P. Rossi, F. Barberis, R. Berti, A. Capelli, and P. G. Sona, *High Temp. High Press.* **19**:371 (1987).
3. L. M. Liang and J. Y. Shi, in *Compendium of the Second Thermophysical Properties Conference of China, Chendu* (1987), pp. 100–104 (in Chinese).
4. Y. Z. Zhang, *Measurement of Thermal Conductivity and Thermal Diffusivity of Rock by Hot Wire and Parallel Wire Method,* M.S. thesis (University of Science & Technology of China, 1988) (in Chinese).
5. W. R. Davis, in *Compendium of Thermophysical Property Measurement Methods,* K. D. Maglic, A. Cezairliyan, and V. E. Peletsky, eds. (Plenum, New York, 1984), pp. 231–259.
6. H. S. Carslaw and J. C. Jaeger, *Conduction of Heat in Solids,* 2nd. Ed. (Oxford, 1986).
7. P. Andersson and G. Backstrom, *Rev. Sci. Instrum.* **47**:205 (1976).
8. Y. S. Touloukian and C. Y. Ho (eds.), *Thermophysical Properties of Matter, Vol. 2. Thermal Conductivity of Nonmetallic Solids; Vol. 5. Specific Heat of Nonmetallic Solids* (Plenum, New York, 1972).
9. Z. S. Chen, H. T. Ni, H. P. Ho, and M. Y. Chen, in *Proceedings of Heat and Mass Transfer Conference of China* (1989), pp. 250–256 (in Chinese).